The pn Junction

- The pn Junction in Equilibrium
  - Built in voltage
  - Carrier concentrations
  - Depletion Region

- The pn Junction Under Bias
  - Carrier Injection

- IV equation for pn junction
  - Minority carrier current in quasi-neutral regions
  - Current flow in depletion region
  - Physical Meaning of $I_0$
  - Reverse breakdown

- Quasi-Fermi Levels

- Non-idealities
  - Series resistance
  - High Injection
  - Depletion Region Recombination
pn Junction in Equilibrium

- Bring p type and n type material together physically
- Thermal equilibrium means no extra heat, no applied voltages, no light
- The electrons from the n type material will diffuse into the p type material (and vice versa for holes)
- After crossing the junction the electrons in the p type (holes in n type) material are minority carriers with a recombination lifetime
- Keep in mind the dopant atoms don’t move – they are part of the crystal
pn Junction in Equilibrium

- Movement of carriers across junction leaves ionized dopant atoms behind
- Material is now charged since the balancing charge carrier is gone
- Electric field is now present around the junction
- Region around the junction is called the space charge region or more commonly the depletion region (since the region surrounding the junction is depleted of carriers)

Note: remember Electric field direction is defined for a positive charge
pn Junction in Equilibrium

- This is a drift current – the E field sweeps holes in n type material to the p type material (minority to majority carriers) and vice versa.
- Unless the minority carrier concentration is increased somehow (heat, optical generation, carrier injection) the drift current will remain low (more on this later).
- Under Equilibrium the nett current is zero i.e. the drift current equals the diffusion current.

Note: remember Electric field direction is defined for a positive charge.
For a system in equilibrium the average energy must be constant (obvious). This also means the Fermi level must be constant.

Away from the junction the original bulk conditions dominate and so the band diagram is unaffected.

Close to the junction the bends bend due to the constant Fermi level, with the bending indicating the strength of the electric field.
Built-in Voltage

- The electric field across the $pn$ junction means a built-in voltage $\Psi_0$ is present.

- Don’t bother trying to measure it with voltmeter, you can’t.

- Built-in voltage is given by the difference between the Fermi levels of the p and n type material.

- Use previously given expressions for carrier concentrations to find expression in terms of doping levels.

$$q\Psi_0 = E_g - E_1 - E_2$$
$$= E_g - kT \ln \left( \frac{N_V}{N_A} \right) - kT \ln \left( \frac{N_C}{N_D} \right)$$
$$= E_g - kT \ln \left( \frac{N_V N_C}{N_A N_D} \right)$$

using
$$n_i^2 = N_c N_v \exp \left( - \frac{E_g}{kT} \right)$$

gives
$$q\Psi_0 = kT \ln \left( \frac{N_C N_V}{n_i^2} \right) - kT \ln \left( \frac{N_C N_V}{N_A N_D} \right)$$
$$\Psi_0 = \frac{kT}{q} \ln \left( \frac{N_A N_D}{n_i^2} \right)$$
Carrier Conc. Under $\equiv_{\text{rm}}$

- Away from the depletion region the carrier concentrations are unaffected.
- Energy difference between Fermi level and conduction (valence) band edge gives electron (hole) concentration – can sketch carrier concentration from band diagram.
- Recall the built-in voltage depends on the doping – so we can relate carrier concentrations to the built-in voltage and each other.

\[
\begin{align*}
 p_{p0} &= p_{p0} \exp\left(\frac{qV}{kT}\right) = N_D \exp\left(\frac{qV}{kT}\right) = \frac{n_i^2}{N_D} \\
n_{p0} &= n_{p0} \exp\left(\frac{qV}{kT}\right) = N_D \exp\left(\frac{qV}{kT}\right) = \frac{n_i^2}{N_A}
\end{align*}
\]
The Depletion Region

- Depletion region surrounds pn junction where carriers have diffused out leaving the ionized dopant atoms.
- Tails off exponentially away from the junction – assuming that it vanishes at some distance from the junction, the depletion region approximation, helps simplify things greatly.
- For constant doping, depletion region approximation means charge density in depletion region is constant (though different on each side of the junction) and zero outside of depletion region.
- The total amount of charge on either side of the junction in the depletion region must be equal.

![Depletion Region Diagram]
Depletion Region Width

- We can find the depletion region width by integrating the charge density over the region to get the electric field

\[
\frac{d\hat{E}}{dx} = \rho = \begin{cases} 
\frac{qN_A}{\varepsilon_S} & \text{when } x_p \leq x < 0 \\
\frac{qN_D}{\varepsilon_S} & \text{when } 0 \leq x < x_n 
\end{cases} \\
N_A x_p = N_D x_n
\]

- The electric field can then be integrated across the region to obtain the built-in voltage

- We already know the built-in voltage in terms of Fermi levels and so we can get the maximum electric field:

\[
\hat{E}_{\text{max}} = -\sqrt{\frac{2q}{\varepsilon} \frac{\psi_0}{\left(\frac{1}{N_A} + \frac{1}{N_D}\right)}}
\]
Depletion Region Width

- Depletion region width is found to be:

\[ W = x_p + x_n = \sqrt{\frac{2\varepsilon}{q} \frac{\psi_o}{\left(\frac{1}{N_A} + \frac{1}{N_D}\right)}} \]

With the two lengths either side of the junction given by:

\[ x_p = W \frac{N_D}{N_A + N_D} \quad \text{and} \quad x_n = W \frac{N_A}{N_A + N_D} \]

Maximum electric field increases with doping and is determined primarily by the lower level of doping.

The lower level of doping has the largest effect on the depletion region width.
pn Junction Under Bias

- Forward bias is a voltage applied to the pn junction that REDUCES the electric field at the barrier, Reverse bias INCREASES the electric field at the junction.

- When bias is applied the balance between drift and diffusion current is destroyed – nett current flow.

- In forward bias, drift current decreases very slightly (can assume it stays the same) but diffusion current increases – Nett current flow.

- In reverse bias opposite occurs with diffusion current decreasing and drift remaining same – Nett current flow (this one is very small).
pn Junction Under Bias

- Band diagram summary of the three types of bias possible

**Equilibrium**

- Depletion region
- \( q\psi_0 \)
- \( E_c \quad E_F \)
- \( E_v \)

**Forward Bias**

- Depletion region
- \( q\psi_0 - V_a \)
- \( E_c \quad E_F \)
- \( E_v \)

**Reverse Bias**

- Depletion region
- \( q\psi_0 + V_a \)
- \( E_c \quad E_F \)
- \( E_v \)

**Current Flow**

- Equilibrium: e-diff., e-drift, hole diff., hole drift
- Forward Bias: e-diff., e-drift, hole diff., hole drift
- Reverse Bias: e-diff., e-drift, hole diff., hole drift

**Nett current**

- Equilibrium: 0
- Forward Bias: +ve
- Reverse Bias: -ve
Depletion Region Under Bias

- Since the applied bias changes the electric field at the junction we can expect the depletion region width to also be changed.
- Assume that the voltage drop is only across the depletion region then for applied bias $V_a$ we have:

\[
\hat{E}_{\text{max}} = -\sqrt{\frac{2\varepsilon \psi_o - V_a}{q \left( \frac{1}{N_A} + \frac{1}{N_D} \right)}}
\]

- This will also affect the depletion region width:

\[
W = x_p + x_n = \sqrt{\frac{2\varepsilon \psi_o - V_a}{q \left( \frac{1}{N_A} + \frac{1}{N_D} \right)}}
\]

- Forward bias decreases the depletion region width whilst reverse bias increases it.
Carrier Injection

• In forward bias the barrier to diffusion current is reduced and so more carriers diffuse across junction from where they are a majority carrier to where they are a minority carrier – minority carrier populations increase

• We can think of this process as carriers being injected from one side of the depletion region to the other – “carrier injection”

• In the absence of generation the concentration of minority carriers decreases away from the junction – recombination

• Carrier concentrations return to equilibrium values more than a few diffusion lengths from the junction
Carrier Injection

• Can assume that the drift and diffusion currents are roughly the same – the nett current is small compared to both, this gives us:

\[ J_p = q \mu_p p \dot{\mathcal{E}} - qD_p \frac{dp}{dx} \approx 0 \]

And it follows that:

\[ \dot{\mathcal{E}} \approx \frac{kT}{q} \frac{dn}{n \; dx} \]

and so

\[ \psi_0 - V_a = -\frac{kT}{q} \ln n_{x_p}^{x_n} = \frac{kT}{q} \ln \left( \frac{n_{nb}}{n_{pa}} \right) \]

• Next we can apply the condition of charge neutrality at the depletion region edges \((x = a \text{ and } x = b)\)

\[ n_{nb} = N_D + p_{nb} \approx n_{n0} = n_{p0} \exp \left( \frac{q\psi_0}{kT} \right) \quad \quad \quad n_{pa} = n_{nb} \exp \left( -\frac{q\psi_0}{kT} \right) \exp \left( \frac{qV_a}{kT} \right) \]

\[ n_{pa} = n_{p0} \exp \left( \frac{qV_a}{kT} \right) = \frac{n_i^2}{N_A} \exp \left( \frac{qV_a}{kT} \right) \]

similarly

\[ p_{nb} = p_{n0} \exp \left( \frac{qV_a}{kT} \right) = \frac{n_i^2}{N_D} \exp \left( \frac{qV_a}{kT} \right) \]
IV equation for \( pn \) junction

- We now have the tools to find the IV equation for a \( pn \) junction

Poisson’s equation

\[
\frac{d\hat{E}}{dx} = \frac{\rho}{\varepsilon} = \frac{q}{\varepsilon} \left( p - n + N_D^+ - N_A^- \right)
\]

Transport Equations

\[
J_n = q\mu_n n\hat{E} + qD_n \frac{dn}{dx} \quad J_p = q\mu_p p\hat{E} - qD_p \frac{dp}{dx}
\]

Continuity Equations

\[
\frac{1}{q} \frac{dJ_n}{dx} = G_n - U_n \quad -\frac{1}{q} \frac{dJ_p}{dx} = G_p - U_p
\]

We assume the following:
1. One dimensional device
2. Thermal equilibrium
3. Steady state
4. Depletion approximation
Minority carriers in quasi-neutral region

- Can make other simplifying assumptions:
  - In quasi-neutral regions charge density is zero, as is the electric field, this means the current flow is purely diffusive
  - No generation, optical or otherwise $G = 0$
  - Low level injection, minority carrier concentration is much lower than the majority carrier concentration
  - Recombination is Shockley Read Hall type, for the moment don’t worry, we just want the form which is
    \[
    \frac{\Delta n}{\tau_n}
    \]
    where $\Delta n$ is excess carrier concentration and $\tau_n$ is minority carrier lifetime

We then obtain

\[
J_n = qD_n \frac{dn(x')}{dx}
\]
\[\text{p side}\]
\[
J_p = -qD_p \frac{dp(x)}{dx}
\]
\[\text{n side}\]

Note: we use two different variables for the distance, $x$ on the n doped side and $x'$ on the p doped side
Minority carriers in quasi-neutral region

- Using the continuity equation we get:

\[ \frac{1}{q} \frac{dJ_n}{dx'} = \frac{\Delta n}{\tau_n} = \frac{n(x') - n_{p0}}{\tau_n} \]

\[ -\frac{1}{q} \frac{dJ_p}{dx} = \frac{\Delta p}{\tau_p} = \frac{p(x) - p_{n0}}{\tau_p} \]

- Combining with the transport equation gives us:

\[ D_n \frac{d^2 n(x)}{dx^2} = \frac{n(x) - n_{p0}}{\tau_n} \]

\[ D_p \frac{d^2 p(x)}{dx^2} = \frac{p(x) - p_{n0}}{\tau_p} \]

- Finally, when we use the following identities \( L_n = \sqrt{D_n \tau_n} \) and \( L_p = \sqrt{D_p \tau_p} \)

- We end up with:

\[ \frac{d^2 \Delta n}{dx^2} = \frac{\Delta n}{L_n^2} \quad \text{and} \quad \frac{d^2 \Delta p}{dx^2} = \frac{\Delta p}{L_p^2} \]

- So the general solutions are

\[ \Delta n = Ae^{\frac{x}{L_n}} + Be^{-\frac{x}{L_n}} \quad \text{and} \quad \Delta p = Ae^{\frac{x}{L_p}} + Be^{-\frac{x}{L_p}} \]
Minority carriers in quasi-neutral region

- Need to solve for $A$ and $B$ using the boundary conditions, assuming surfaces of the $pn$ junction are far away

  1. Boundary condition at depletion region edge ($x = 0$)

  \[ n(0) = \frac{n_i^2}{N_D} \exp\left(\frac{qV_0}{kT}\right) \]

  2. Boundary condition at “surface”, carrier concentration must be finite, this forces $A$ to be zero

- This gives us the following exact solutions:

  \[ p_n(x) = p_n^0 + p_n^0 \left[ \exp\left(\frac{qV}{kT}\right) - 1 \right] e^{-\frac{x}{L_p}} \]

  \[ n_p(x) = n_p^0 + n_p^0 \left[ \exp\left(\frac{qV}{kT}\right) - 1 \right] e^{-\frac{x}{L_n}} \]

- Notes: condition 1. will apply to most devices, the boundary condition at the surface 2. can be generation or current as well as carrier concentration
Minority carriers in quasi-neutral Region

- Remember we are assuming charge neutrality away from the depletion region – majority carrier concentration goes up to match minority carrier concentration.
- Also keep in mind the carrier concentrations ‘die off’ due to recombination as we move away from the depletion region.
Minority carriers in quasi-neutral Region

- Knowing how the carrier concentrations vary over distance allows us to find the current e.g.

\[ J_p = -qD_p \frac{dp}{dx} \quad J_n = qD_n \frac{dn(x')}{dx} \]

- We find the current by only looking at the change in minority carrier concentration (more on this later)

- Easy enough to plug expressions from earlier in and get the following:

\[ J_p = q \frac{D_n n_p 0}{L_n} \left[ \exp\left(\frac{qV}{kT}\right) - 1 \right] e^{-\frac{x}{L_n}} \]

\[ J_n = -q \frac{D_p P_n 0}{L_p} \left[ \exp\left(\frac{qV}{kT}\right) - 1 \right] e^{-\frac{x}{L_p}} \]
Current flow in depletion region

- Difficult to calculate since the electric field is present
- Since depletion width is typically much smaller than diffusion length of minority carriers can assume recombination is zero in depletion region since carriers are swept through by the electric field
- Any non-idealities mean we need to find an expression for the recombination
- Since we have no generation or recombination we have

\[
\frac{1}{q} \frac{dJ_n}{dx} = G_n - U_n = 0
\]

\[
- \frac{1}{q} \frac{dJ_p}{dx} = G_p - U_p = 0
\]

and so the current for each carrier is a constant
Total Current

- We have the minority carrier current and the current through the depletion region but don’t have an easy way of finding majority carrier current.
- Don’t need to find since the current flowing through the pn junction structure must be constant to avoid charging.
- So, in reality we only need to find the two minority carrier currents at the edges of the depletion region.

\[
J_{\text{total}} = J_n(x' = 0) + J_p(x = 0)
\]

\[
= \left[ q \frac{D_n n_{p0}}{L_n} + q \frac{D_p p_{n0}}{L_p} \right] \left[ e^{\frac{qV}{kT}} - 1 \right]
\]

\[
= \left[ q \frac{D_n n_i^2}{L_n N_A} + q \frac{D_p n_i^2}{L_p N_D} \right] \left[ e^{\frac{qV}{kT}} - 1 \right]
\]
Meaning of $I_0$

- Ideal diode equation
  \[ I = I_0 \left[ \exp \left( \frac{qV}{kT} \right) - 1 \right] \]
  where $I_0$ is
  \[ I_0 = qA \left( \frac{D_n n_i^2}{L_n N_A} + \frac{D_p n_i^2}{L_p N_D} \right) \]

- Forward bias current depends on how many carriers are at the edge of the depletion region, this is determined by diffusivity, recombination and background minority carrier concentration
- Increasing recombination or diffusivity will increase current
- More heavily doped side dominates the current flow
- The forward bias current is a recombination current
- Higher band gap will give a lower recombination current
Reverse Breakdown

- Reverse bias increases electric field at junction and hence the diffusion current is lowered.
- Drift current determined by the number of carriers being swept across the junction – so drift current doesn’t increase significantly.
- Drift current does increase slightly, however, due to the depletion region expanding and more carriers being thermally generated there, hence more current.
- If reverse voltage is high enough we get tunneling.
Quasi-Fermi Levels

- In Thermal equilibrium the Fermi level was given as the average energy of the carriers (doesn’t change in equilibrium)
- In steady state a quasi-Fermi level can be defined to describe the carrier concentrations and device behavior analogously

\[ n = N_C \exp[(E_F - E_C)/kT] \quad p = N_V \exp[(E_V - E_{Fp})/kT] \]

- Quasi Fermi level separation is a measure of the disturbance from equilibrium with the separation being close to the bias (not always equal)

\[ np = n_i^2 \exp[(E_{Fn} - E_{Fp})/kT] \]

- Since they measure the carrier concentration they relate to the current present in the following way:

\[ J_n = \mu_n n \frac{dE_{Fn}}{dx} \quad J_p = \mu_p n \frac{dE_{Fp}}{dx} \]
Quasi-Fermi Levels

- Constant across the depletion region
- Vary smoothly and gradually, only get sharp changes at interfaces

See above for a pn junction under forward bias in the dark

Note that the quasi – Fermi levels come back together away from the depletion region because of recombination of carriers
Series Resistance

- So far no resistance assumed but in reality there is always resistance for current.
- Series resistance consists of the resistance of the quasi-neutral bulk regions and the contact resistance (metal/sc).
- Voltage across junction is lower than that between contacts – current is lower than expected, becoming worse as current increases.

\[
I = I_0 \left[ \exp \left( \frac{q(V - IR_s)}{kT} \right) - 1 \right]
\]
High Injection

- High injection is when the minority carrier concentration is close to the majority carrier concentration – only low injection considered in derivation of IV equation.

- High injection comes about for high forward biases and will occur in the more lightly doped side first.

\[ p_{nb} = \frac{n_i^2}{N_D} \exp\left(\frac{qV_{hi}}{kT}\right) = N_D \]
\[ V_{hi} = \frac{2kT}{q} \ln\left(\frac{N_D}{n_i}\right) \]

- Recombination no longer determined solely by minority carrier concentration, current is now:

\[ I = I_S \exp\left(\frac{qV}{2kT}\right) \]

- There are other recombination mechanisms present so current should be written:

\[ I = I_0 \left[ \exp\left(\frac{qV}{n kT}\right) - 1 \right] \]

where \( n \) is between 1 and 2. 2 represents high injection.
Depletion Region Recombination

- Recombination in the depletion region previously ignored but will be dominant for low current.
- Since the $n$ and $p$ concentrations have to ‘cross’ since they are shifting from majority carrier to minority carrier.
- Recombination will be maximum where the two carrier populations are equal since in general $R = B.n.p$ where $B$ is a constant.
- A recombination ‘sweet spot’ will exist near the centre of the depletion region.
- Since the current is a recombination current the IV curve will be changed.
Depletion Region Recombination

- The current due to depletion region recombination is given by:

\[ I = I_{\text{rec}} \exp \left( \frac{qV}{2kT} \right) \]

\[ I_{\text{rec}} \approx \frac{qA n_i^2 W}{2 \tau_e} \]

\[ \tau_e t = \frac{1}{2} \left( \tau_{n0} + \tau_{p0} \right) \]

Forward bias

Reverse bias